

## Study and Comparison of Approximate String Matching Algorithms

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### **Abstract :**

*Many database applications to support similarity based retrieval on stored multimedia objects are of increasing interest in the sector. This paper demonstrates three approximate string matching algorithms. These are the Brute force, Lipschitz Embeddings and Ball Partitioning algorithms. While Brute Force performs approximate string matching based on distance measures of the query object from each string stored in the database, Lipschitz Embeddings uses a far more efficient approach which embeds the stored strings in database in vector space so that the distances of embedded strings approximates the actual distances. Ball Partitioning algorithm, much more efficient than Brute force but less efficient than Lipschitz algorithm, performs search in approximate string matching based on distances where queries operate on an arbitrary search hierarchy. The paper compares and makes an analysis of the three algorithms which are suitable for approximate matching of strings stored in database text files, an issue much required in the context of similarity based retrieval of objects. The work can be extended for future work by taking into account a larger number of algorithms suited to approximate string matching for the benefit of a wider scope of comparisons and picking out the most optimal one .*

**Keywords :** *Edit distance, Brute force, Lipschitz embeddings, Ball Partitioning, Similarity based retrieval*

### **I. INTRODUCTION**

Finding the occurrences of a given query string (pattern) from a possibly very large text is an old and fundamental problem in computer science. It emerges in applications ranging from text processing and music retrieval to bioinformatics. This task, collectively known as string matching, has several different variations. The most natural and simple of these is exact string matching, in which, like the name suggests, one wishes to find only occurrences that are exactly identical to the pattern string. This type of search, however, may not be adequate in all applications if for example the pattern string or the text may contain typographical errors.

Perhaps the most important applications of this kind arise in the field of bioinformatics, as small variations are fairly common in DNA or protein sequences. The field of approximate string matching, which has been a research subject since the 1960's, answers the problem of small variation by permitting some error between the pattern and its occurrences. Given an error threshold and a metric to measure the distance between two strings, the task of approximate string matching is to find all substrings of the text that are within (a distance of) the error threshold from the pattern.

In this paper we concentrate on approximate string matching that uses so called unit-cost edit distance as the metric to measure the distance between two strings. One possible definition of the approximate string matching problem is the following: Given a pattern string  $P = p_1p_2\dots p_m$  and a text string,  $T = t_1t_2\dots t_n$  find a substring  $T_j = t_j\dots t_{j+l-1}$  in  $T$ , which, of all substrings of  $T$ , has the smallest edit distance to the pattern  $P$ . The most common application of approximate matchers until recently has been spell checking. With the availability of large amounts of DNA data, matching of nucleotide sequences has become an important application. Approximate matching is also used to identify pieces of music from small snatches and in spam filtering.[1]

This paper demonstrates three different approximate string matching algorithms: Brute Force algorithm for approximate string matching, Lipschitz Embeddings Algorithm, Ball Partitioning Algorithm elaborately and a comparison among these algorithms is illustrated. A concise overview of edit distance is discussed also.

## II. RELATED WORKS

A number of researchers have presented variations on approximate string matching algorithms. Luis M. S. Russo, Gonzalo Navarro, Arlindo L. Oliveira, and Pedro Morales focused on indexed approximate string matching [2]. They studied approximate string matching algorithms for Lempel- Ziv compressed indexes and for compressed suffix trees/arrays. Lempel-Ziv indexes split the text into a sequence of so-called phrases of varying length. They are efficient to find the (exact) occurrences that lie within phrases, but those that span two or more phrases are costlier. Luis M. S. Russo, Gonzalo Navarro, Arlindo L. Oliveira , and Pedro Morales started by adapting the classical method of partitioning into exact search to self-indexes, and optimized it over a representative of either class of self-index. Then, they showed that a Lempel-Ziv index can be seen as an extension of the classical q-samples index and they improved hierarchical verification to extend the matches of

pattern pieces to the left or right which largely reduced the accesses to the text, which are expensive in self-indexes. Zheng Liu, James Borneman , Tao Jiang presented a fast algorithm for approximate string matching called FFAST [3]. It aimed at solving a popular variant of the approximate string matching problem, the  $k$ -mismatch problem, whose objective is to find all occurrences of a short pattern in a long text string with at most  $k$  mismatches. FFAST generalizes the well-known Tarhio-Ukkonen's  $k$ -mismatches algorithm. In the Tarhio-Ukkonen algorithm, the shift distance is calculated as the minimum one such that there exists at least one match when aligning the rightmost  $k+1$  text characters in the current alignment with the pattern after a shift. In order to achieve faster matching process, FFAST instead calculates the shift distance as the minimum one such that the rightmost  $k+x$  characters of the current aligned text will have at least  $x$  matches after the shift. Here,  $x$  generally takes a small integer value, e.g., two or three. Theoretically, they proved that FFAST on average skips more characters than the Tarhio- Ukkonen algorithm in a single shift, and makes fewer character comparisons in an entire matching process. Bit-vector algorithm of Myers is one of the most notable recent algorithms in the area of approximate string matching algorithms. The main idea of this algorithm is to parallelize the dynamic programming matrix by using bit-vectors to encode the list of  $m$  (arithmetic) differences between successive entries in a column of the dynamic programming matrix. Heikki Hyyro, Kimmo Fredriksson, Gonzalo Navarro explored different ways to increase the bit-parallelism for approximate string matching by modifying the bit-vector algorithm of Myers when the pattern is short or if the maximum number of differences permitted is moderate with respect to the alphabet size [4]. They showed how multiple patterns can be packed in a single computer word so as to search for multiple patterns simultaneously. They showed two ways to do this. The first one permits searching for several patterns simultaneously. The second one boosts the search for a single pattern by processing several text positions simultaneously. William I. Chang and Eugene L. Lawlers' research area was "Approximate String Matching in Sublinear Expected Time" [ 5]. They defined the approximate substring matching problem and gave efficient algorithms based on their techniques. Special cases include several applications to genetics and molecular biology. For example, even allowing errors, they found long common blocks of the text and pattern (local similarities), or selected from among a set of text fragments ones that overlap one end of the pattern (sequence assembly). These are common tasks in DNA sequence analysis. Gonzalo Navarro researched on "A Guided Tour to Approximate String Matching"[6] where this work focused on the problem of string matching that allows errors, also called approximate string matching. The general goal is to perform string matching of a pattern in a text where one or both of them have suffered some kind of (undesirable) corruption. Some examples are recovering the original signals after their transmission over noisy channels, finding DNA subsequences after possible mutations, and text searching where there are typing or spelling errors.

### III. EDIT DISTANCE AND APPROXIMATE STRING MATCHING

The edit distance  $ed(P,T)$  between the strings P and T is defined in general as the minimum cost of any sequence of edit operations that edits P into T or vice versa. Differing in their choices of the allowed set of edit operations and their costs, for example the following types of edit distance have appeared in the literature:

- Levenshtein edit distance
- Damerau edit distance
- Weighted/generalized edit distance
- Hamming distance
- Longest common subsequence

Approximate string matching is closely related to edit distance. It refers to searching for approximate matches of a pattern string P from a usually much longer text string T, where edit distance is used as a measure of similarity between P and the substrings of T. [7]

The work in this paper concentrates on Levenshtein edit distance in which the allowed edit operations are insertion, deletion or substitution of a single character, and each operation has the cost 1. This type of edit distance is sometimes called unit-cost edit distance. Levenshtein edit distance is perhaps the most common form of edit distance, and often the term edit distance is assimilated to it. [8]

The following algorithm of Levenshtein edit distance fills the (integer) entries in a matrix m whose two dimensions equal the lengths of the two strings (s1,s2) whose edit distances is being computed; the (i,j) entry of the matrix will hold (after the algorithm is executed) the edit distance between the strings consisting of the first i characters of s1 and the first j characters of s2. The central dynamic programming step is depicted in Lines 8-10 , where the three quantities whose minimum is taken correspond to substituting a character in s1, inserting a character in s1 and inserting a character in s2 [9] :

EDIT\_DISTANCE (S1, S2)

1. int  $m[i,j] = 0$
2. for  $i= 1$  to  $|S1|$
3. do  $m[i,0] = i$
4. for  $j = 1$  to  $|S2|$
5. do  $m[0,j] = j$
6. for  $i= 1$  to  $|S1|$
7. do for  $j = 1$  to  $|S2|$ 
  8. do  $m[ i, j ] = \min \{ m[i-1,j-1] + \text{if } (S1[i] = S2 [j])$   
then 0 else 1,

9.  $m[i-1, j] + 1$ ,
10.  $m[i, j-1] + 1$  }
11. return  $m[|S1|, |S2|]$

Fig 1: Levenshtein edit distance algorithm for computing the edit distance between strings  $s1$  and  $s2$

#### IV. BRUTE FORCE ALGORITHM FOR APPROXIMATE STRING MATCHING

A brute-force approach would be to compute the edit distances to query object( $q$ ) from all  $N$  substrings of text( $T$ ), and then choose the substring with the minimum distance. The sequential steps are given below:

Steps:

1. Read  $N$  ( $=50$ ) strings from the text file  $T$ .
2. Take an input (query object  $q$ ) from the user.
3. Calculate the edit distances to query object,  $q$  from all  $N$  strings of  $T$ .
4. Find out the minimum edit distance.
5. Output will be the string which has the minimum edit distance.

Let  $N=5$  and  $T$  is: {been, bid, moon, sun, star}

Say,  $q='seen'$

The edit distances to query object  $q$  from all  $N$  strings of  $T$  will be respectively:

{1,4,3,2,3}

The minimum distance is: 1 Hence the output will be: been

#### V. LIPSCHITZ EMBEDDINGS ALGORITHM

A Lipschitz embedding is defined in terms of a set  $R$  of subsets of  $S$ (the set of objects),  $R=\{A1, A2, \dots, Ak\}$ . The subsets  $A_i$  are termed the reference sets of the embedding. Let  $d(o, A)$  be an extension of the distance function  $d$  to a subset  $A$  of  $S$ , such that  $d(o, A) = \min\{d(o, x)\}$ , where  $x$  is an element of  $A$ . An embedding with respect to  $R$  is defined as a mapping  $F$  such that  $F(o) = (d(o, A1), d(o, A2), \dots, d(o, Ak))$ .

To elaborate on how a query is implemented, suppose that we want to find the nearest object to a query object  $q$ . We first determine the point  $F(q)$  corresponding to  $q$ . Next, we examine the objects in the order of their distances from  $F(q)$  in the embedding space. When using a multidimensional index, this can be achieved by using an incremental nearest neighbor algorithm. Suppose that point  $F(a)$  corresponding to object  $a$  is the closest point to  $F(q)$  at a distance of  $\partial(F(a), F(q))$ . We compute the distance  $d(a, q)$  between the

corresponding objects. At this point, we know that any object  $x$  with  $\partial(F(x), F(q)) > d(a, q)$  cannot be the nearest neighbor of  $q$  since the contractive property then guarantees that  $d(x, q) > d(a, q)$ . Therefore,  $d(a, q)$  now serves as an upper bound on the nearest neighbor search in the embedding space. We now find the next closest point  $F(b)$  corresponding to object  $b$ , subject to our distance constraint  $d(a, q)$ .

If  $d(b, q) < d(a, q)$ , then  $b$  and  $d(b, q)$  replace object  $a$  and  $d(a, q)$  as the current closest object and upper bound distance, respectively; otherwise,  $a$  and  $d(a, q)$  are retained. This search continues until encountering a point  $F(x)$  with  $\partial(F(x), F(q)) > d(y, q)$ , where  $y$  is the current closest object which is now guaranteed to be the actual closest object to  $q$ . [10]

**Algorithm:**

**Input :** A text file  $T$  containing  $N$  strings ( $O_1, O_2, \dots, O_N$ ) and a query object  $q$ .

**Output :** Nearest string to  $q$  with corresponding edit distance.

**Steps:**

1. Construct a text file  $R$  of  $k$  strings ( $A_1, A_2, \dots, A_k$ ) by choosing randomly from  $N$  strings of  $T$ .
2. Compute  $F(q)$ , the array of edit distances of the query object  $q$  to  $k$  strings of  $R$ , that is,  $F(q) = (d(q, A_1), d(q, A_2), \dots, d(q, A_k))$  where,  $d(q, A_j)$  is the edit distance between  $q$  and  $A_j$ . Here,  $1 \leq j \leq k$ .
3. Compute  $F(O_i)$ , the array of edit distances of each string  $O_i$  in  $T$  to  $k$  reference strings in  $R$ . that is,  
 $F(O_i) = (d(O_i, A_1), d(O_i, A_2), \dots, d(O_i, A_k))$ .  
Here,  $1 \leq i \leq N$ .
4. Calculate  $\partial(F(O_j), F(q))$ , the distance between each string  $O_j$  of  $T$  to query string  $q$  in embedding space where,

$$\partial(F(O_j), F(q)) = \sum_{i=1}^k \left( \frac{|d(O_j, A_i) - d(q, A_i)|}{k^{1/p}} \right)^{1/p}$$

where  $1 \leq j \leq N$ ,  $1 \leq i \leq k$ ,  $p=2$ .

5. Find the minimum value among  $\partial(F(O_i), F(q))$ , if the minimum is  $\partial(F(O_m), F(q))$ , then find  $d(O_m, q)$ . ( $1 \leq i \leq N$ ).
6. For  $i=1$  to  $N$ ,  
if  $(\partial(F(O_i), F(q)) > d(O_m, q))$

then edit\_distance= d(Om ,q) and nearest\_string= Om.

else if (d(Oi ,q) < d(Om ,q))

then edit\_distance= d(Oi ,q) and nearest\_string = Oi

7 . Show the edit\_distance and nearest\_string.

## VI. BALL PARTITIONING ALGORITHM

The vp-tree (Vantage Point Tree) uses ball partitioning (and thus is a variant of the metric tree. In this method, we pick a pivot p randomly from S containing 50 strings/objects (p is termed as a vantage point; compute the median r of the distances of the other objects to p, and then divide the remaining objects into roughly equal sized subsets S1 and S2 as follows

$$S_1 = \{o \in S \setminus \{p\} \mid d(p, o) < r\}, \quad \text{and} \\ S_2 = \{o \in S \setminus \{p\} \mid d(p, o) \geq r\}.$$

Thus, the objects in S1 are inside the ball of radius r around p, while the objects in S2 are outside this ball. Applying this rule recursively leads to a binary tree, where a pivot object is stored in each internal node, with the left and right sub trees corresponding to the subsets inside and outside the corresponding ball, respectively. In the leaf nodes of the tree we would store one or more objects, depending on the desired capacity.

**Pivot Selection :** Pivot is chosen randomly in this algorithm and in the vp-tree, the ball radius is always chosen as the median so that the two subsets are roughly equal in size.

**Search :** We visit the left child if and only if

$\max\{d(q, p)-r, 0\} \leq \epsilon$  and the right child if and

only if  $\max\{r-d(q, p), 0\} \leq \epsilon$

Definitions of some keywords:

Radius of pivot- r

Query object- q

Pivot object- p

Edit distance of pivot and query object-  $\epsilon$

Another notation of edit distance between pivot and query  $d\{q, p\}$  also. [11]

The algorithm is divided into two parts:

**Creating VP-TREE:**

1. Read N (=50) strings from a text file.
2. Select a pivot (p) randomly from N strings.
3. Compute edit-distance (d) of pivot from the remaining strings.
4. Values of edit-distances (from pivot to others) will be kept in an array.
5. Sort the values of the array and find out median using formula of median.
6. The median is the radius (r) of pivot. A flag value will be increased.
7. Take this pivot and radius as the input of VP-tree's first node.
8. Similarly, repeat steps 2-6 and find out pivot for each step; if the pivot's radius is less than its parent node's radius r and  $\geq 0$ , then put it on left node and if  $\geq r$  then put it on right node. In step 2 each time N decreases by 1.
9. This process will continue until each internal node ends at leaf with a child or we can keep a cluster of strings at each leaf.

**Searching Phase:**

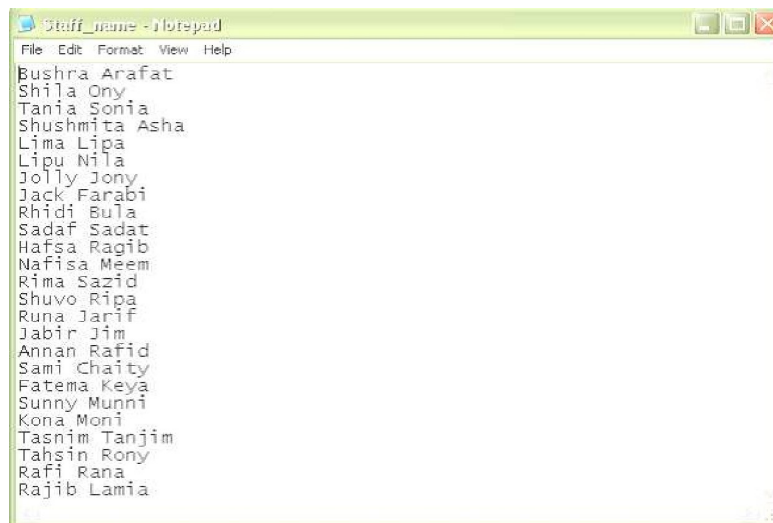
1. Take an input of query object (q).
2. When a value will be assigned in VP-tree (pivot and radius) then find the edit distance of pivot and query object. And check if,  
 $\text{Max } \{d(q, p) - r, 0\} \leq E$ , then visit left node, otherwise not.  
 $\text{Max } \{r - d(q, p), 0\} \leq E$ , then visit right node, otherwise not.  
E= the edit-distance between pivot and query object.
3. Keep the edit-distance and the pivot in an array of structure.
4. Repeat the steps of searching phase 1-3 until we get child of each node.
5. Now find the minimum edit distance of all edit distances, and the corresponding pivot which will be the nearest neighbor of query object.

VII. ANALYSIS

Lipschitz Embedding Algorithm uses a straight forward procedure to match an approximate string to the query string. It creates reference strings, uses mathematical formula to find out minimum edit distance and



the corresponding nearest string to the query string. Ball Partitioning Algorithm creates a VP tree and visits left and right nodes of the tree to find minimum edit distance and the corresponding string which is nearest to the query string. Execution time increases with the increase of visited nodes. Brute Force Algorithm for approximate string matching uses most inefficient way to find out nearest string. It calculates edit distances of all the text strings from the query string and then checks all the distances to find out the minimum distance. In Lipschitz Embeddings and Ball Partitioning Algorithm, this checking (like Brute Force Algorithm- “checks all the distances”) is not applicable. Both of them follow some mathematical procedures, to find the expected result. As a result, among the three approximate string matching algorithms, the fastest is Lipschitz Embeddings Algorithm, then the Ball Partitioning Algorithm and the slowest is Brute Force Algorithm for approximate string matching. In order to evaluate the practical performances of approximate string matching algorithms, we have implemented them in C in a homogeneous way on the same text file (Staff\_name.txt) having 50 strings to make the comparisons significant. Here, the Staff\_name text file has been extracted from a data-table of a database management system.



*Fig 1: Snapshot of the Staff\_name.txt*

Snapshots of the results of algorithms applied on Staff\_name.txt file are given below according to their execution time (from the fastest to the slowest):

**a) Lipschitz Embeddings Algorithm**

```
C:\VCB\BIN\NALI-1.EXE
Please enter the text file name,I: Staff_name.txt

Text strings are:
Bushra Arafat Shila Ony Tania Sonia Shushmita Asha Lima Lipa Lipu N
ila Jolly Jony Jack Farabi Rhidi Bula Sadaf Sadat Hafsa Ragib Mafisa
Meem Rima Sazid Shuvo Ripa Runa Jarif Jabir Jim Annan Rafid Sami Ch
aity Fatema Meva Sunny Munni Kona Moni Tasnim Tanjim Tahsin Rony Rafi
Rana Rajib Lania

Randomly selected reference strings,R:
Sami Tanjim Arafat Munni Kona Sadat Meem Jim Bushra Jony

Now enter the query object,q: Sadam

Nearest String: Sadaf
Edit distance: 1
Execution time: 21 sec
```

b) Ball Partitioning Algorithm

```
C:\VCB\BIN\NALI-1.EXE
Give the filename for files : Staff_name.txt

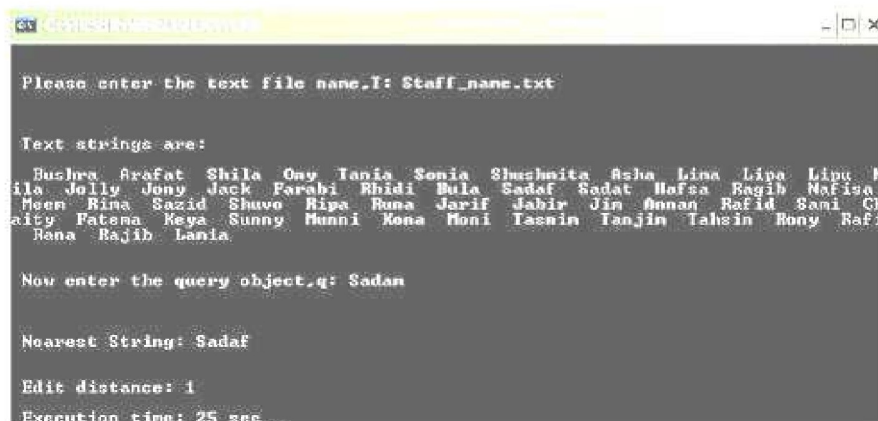
first node : Hafsa
left of first node--Nafisa
Nafisa--left--Rafi
Nafisa--right--Rafid
Rafi--left--Tania
Rafi--right--Rana
Rafid--left--Lamia
Rafid--right--
Tania--left--
Tania--right--
Rana--left--
Rana--right--
Lamia--left--
Lamia--right--

right of first node--Shushmita
Shushmita--left--Bushra
Shushmita--right--Jony
Bushra--left--Shila
Bushra--right--Shuvo
Jony--left--Ony
Jony--right--Meem
Shila--left--Sonia
Shila--right--Sami
Shuvo--left--Rony
Shuvo--right--Jolly
Ony--left--Lima
Ony--right--Arafat
Meem--left--
Meem--right--
Shila or Sami--right node :      Asha
Shuvo or Jolly--right node :    Chaity
Ony or Arafat--right node :      Moni
Meem or --left node :           Ripa      Kona      Sunny      Jim      Runa      Jack
Meem or --right node :          Munn      Keya      Fatema     Bula     Lipa
Meem or --left node :           Annan   Sadat     Sadaf
Meem or --right node :          Rajib   Tahsin   Tanjim     Tasnim   Jabir     Jarif     Sazid
Ragib Rhidi Farabi

Give the query object(string) : Sadam

Nearest neighbor of Sadam is Sadaf and distance: 1
Execution time: 22 sec
*****
```

c) Brute Force algorithm for approximate string match



```
Please enter the text file name: Staff_name.txt

Text strings are:
Bushra Arafat Shila Ony Tania Sonia Shushmita Asha Lina Lipa Lipu N
ila Jilly Jony Jack Farabi Bhidi Bula Sadaf Sadat Hafsa Ragib Rafisa
Heen Rina Sazid Shuvo Ripsa Runa Jarif Jabir Uin Anwar Rafid Sami Gh
airy Fatema Keya Sunny Munni Kona Moni Tasnin Tanjin Tahsin Rony Rafi
Rana Rajib Lania

Now enter the query object: q: Sadan

Nearest String: Sadaf

Edit distance: 1

Execution time: 25 sec
```

## VIII. CONCLUSION AND FUTURE WORK

In this paper we have presented some existing approximate string matching algorithms, explored their characteristics and implemented them in C. As we have seen, their execution times are different. Lipschitz Embeddings Algorithm is faster than Ball Partitioning and Brute Force Algorithm. On the other hand Ball Partitioning Method needs too much memory than the others. We have used real life data and converted database tables into text files and applied our approximate string matching codes on these text files.

There are many other advanced approximate string matching algorithms having different characteristics and usefulness which can be applied for wider comparisons. We have worked on text files having 50 strings each, which can be expanded by including more strings about 1000 strings or more. In Ball Partitioning Method, clusters of strings can be represented at the leaves for the sake of saving the storage space. These methods can be applied for similarity search which is in a leading role in multimedia databases and other database applications involving complex objects.

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